New Version of the Intercriteria Analysis

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Abstract. Intercriteria Analysis (ICA) is based on the relations between the evaluations of some given objects by some fixed criteria. The new version of ICA is based on the comparison between the differences of the evaluation of the objects with a fixed threshold.
Keywords. Data, Intercriteria analysis, Index matrix, Intuitionistic fuzzy pair
AMS Classification. 03E72

1 Introduction

The concept of InterCriteria Analysis (ICA) was introduced in [6]. It is based on the apparatus of the Index Matrices (IMs, see [1,3,9]) and of Intuitionistic Fuzzy Sets (IFSs, see, e.g., [2]). During last years a lot of papers over the theory and applications of ICA were published (see [8]).

Here, for the first time we discuss the idea to compare the differences of the evaluation of the objects with a fixed threshold. In a result we will obtain Intuitionistic Fuzzy Pairs (IFP, see [4,7]), determining the nearness between the criteria.

2 Short notes on intuitionistic fuzzy pairs

The Intuitionistic Fuzzy Pair (IFP, see [4,7]) is an object in the form $\langle a, b\rangle$, where $a, b \in [0, 1]$ and $a + b \leq 1$, that is used as an evaluation of some object or process and which components (a and b) are interpreted as degrees of membership and non-membership, or degrees of validity and non-validity, or degree of correctness and non-correctness, etc. One of the geometrical interpretations of the IFPs is shown on Fig. 1.

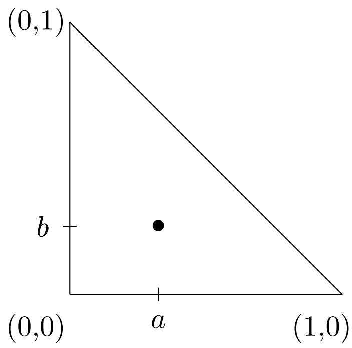

Fig. 1.

Let us have two IFPs $x = \langle a, b\rangle$ and $y = \langle c, d\rangle$. We define the relations

$$\begin{array}{lll} x < y & \text{iff} & a < c \text{ and } b > d \\ x > y & \text{iff} & a > c \text{ and } b < d \\ x \geq y & \text{iff} & a \geq c \text{ and } b \leq d \\ x \leq y & \text{iff} & a \leq c \text{ and } b \geq d \\ x = y & \text{iff} & a = c \text{ and } b = d \end{array}$$

3 Short remarks on index matrices

The concept of Index Matrix (IM) was discussed in a series of papers collected in [1,3].

Let I be a fixed set of indices and $\mathcal{R}$ be the set of the real numbers. By IM with index sets K and L $(K, L \subset I)$, we denote the object:

$$[K, L, \{a_{k_i,l_j}\}] \equiv \begin{array}{c|cccc} & l_1 & l_2 & \ldots & l_n \\ \hline k_1 & a_{k_1,l_1} & a_{k_1,l_2} & \ldots & a_{k_1,l_n} \\ k_2 & a_{k_2,l_1} & a_{k_2,l_2} & \ldots & a_{k_2,l_n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ k_m & a_{k_m,l_1} & a_{k_m,l_2} & \ldots & a_{k_m,l_n} \end{array},$$

where $K = \{k_1, k_2, ..., k_m\}$, $L = \{l_1, l_2, ..., l_n\}$, for $1 \leq i \leq m$, and $1 \leq j \leq n : a_{k_i,l_j} \in \mathcal{R}$.

In [1,3], different operations, relations and operators are defined over IMs. For the needs of the present research, we will introduce the definitions of some of them.

When elements a_{k_i,l_j} are some variables, propositions or formulas, we obtain an extended IM with elements from the respective type. Then, we can define the evaluation function V that juxtaposes to this IM a new one with elements – IFPs $\langle \mu, \nu \rangle$, where $\mu, \nu, \mu + \nu \in [0, 1]$. The new IM, called Intuitionistic Fuzzy IM (IFIM), contains the evaluations of the variables, propositions, etc., i.e., it has the form

$$V([K, L, \{a_{k_i,l_j}\}]) = [K, L, \{V(a_{k_i,l_j})\}] = [K, L, \{\langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle\}]$$

$$= \begin{array}{c|ccccc} & l_1 & \dots & l_j & \dots & l_n \\ \hline k_1 & \langle \mu_{k_1,l_1}, \nu_{k_1,l_1} \rangle & \dots & \langle \mu_{k_1,l_j}, \nu_{k_1,l_j} \rangle & \dots & \langle \mu_{k_1,l_n}, \nu_{k_1,l_n} \rangle \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ k_i & \langle \mu_{k_i,l_1}, \nu_{k_i,l_1} \rangle & \dots & \langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle & \dots & \langle \mu_{k_i,l_n}, \nu_{k_i,l_n} \rangle \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ k_m & \langle \mu_{k_m,l_1}, \nu_{k_m,l_1} \rangle & \dots & \langle \mu_{k_m,l_j}, \nu_{k_m,l_j} \rangle & \dots & \langle \mu_{k_m,l_n}, \nu_{k_m,l_n} \rangle \end{array},$$

where for every $1 \le i \le m, 1 \le j \le n$: $V(a_{k_i,l_j}) = \langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle$ and $0 \le \mu_{k_i,l_j}, \nu_{k_i,l_j}, \mu_{k_i,l_j} + \nu_{k_i,l_j} \le 1$.

4 The new version of Intercriteria analysis

Now, following and modifying [6], we describe the new version of ICA.

Let us have the set of objects $O = \{O_1, O_2, ..., O_n\}$ that must be evaluated by criteria from the set $C = \{C_1, C_2, ..., C_m\}$.

Let us have an IM

$$A = \begin{array}{c|ccccccc} & O_1 & \cdots & O_i & \cdots & O_j & \cdots & O_n \\ \hline C_1 & a_{C_1,O_1} & \cdots & a_{C_1,O_i} & \cdots & a_{C_1,O_j} & \cdots & a_{C_1,O_n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ C_k & a_{C_k,O_1} & \cdots & a_{C_k,O_i} & \cdots & a_{C_k,O_j} & \cdots & a_{C_k,O_n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ C_l & a_{C_l,O_1} & \cdots & a_{C_l,O_i} & \cdots & a_{C_l,O_j} & \cdots & a_{C_l,O_n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ C_m & a_{C_m,O_1} & \cdots & a_{C_m,O_i} & \cdots & a_{C_m,O_j} & \cdots & a_{C_m,O_n} \end{array},$$

where for every p, q $(1 \le p \le m,\ 1 \le q \le n)$:

(1) C_p is a criterion, taking part in the evaluation,
(2) O_q is an object, being evaluated.
(3) a_{C_p,O_q} is a real number that represents the evaluations of the q-th object by the p-th criterion,
(4) ε_p is a fixed threshold for the p-th criterion.

For example, ε_p can have the form

$$\varepsilon_p = \omega(\max_{1 \le q \le m} a_{C_p,O_q} - \min_{1 \le q \le m} a_{C_p,O_q}),$$

where ω can be equal for all $\varepsilon_1, \ldots, \varepsilon_m$.

Let $S^{\mu}_{k,l}$ be the number of cases in which

$$|a_{C_k,O_i} - a_{C_k,O_j}| < \varepsilon_k$$

and

$$|a_{C_l,O_i} - a_{C_l,O_j}| < \varepsilon_l.$$

Let $S^{\nu}_{k,l}$ be the number of cases in which

$$|a_{C_k,O_i} - a_{C_k,O_j}| > \varepsilon_k$$

or

$$|a_{C_l,O_i} - a_{C_l,O_j}| > \varepsilon_l.$$

Let $S^{\pi}_{k,l}$ be the number of cases in which

$$|a_{C_k,O_i} - a_{C_k,O_j}| = \varepsilon_k$$

or

$$|a_{C_l,O_i} - a_{C_l,O_j}| = \varepsilon_l.$$

Obviously,

$$S^{\mu}_{k,l} + S^{\nu}_{k,l} + S^{\pi}_{k,l} = \frac{n(n-1)}{2}.$$

Now, for every k, l, such that $1 \le k, l \le m$ and for $n \ge 2$, we define

$$\mu_{C_k,C_l} = 2\frac{S^{\mu}_{k,l}}{n(n-1)}, \quad \nu_{C_k,C_l} = 2\frac{S^{\nu}_{k,l}}{n(n-1)}.$$

Hence,

$$\mu_{C_k,C_l} + \nu_{C_k,C_l} = 2\frac{S^{\mu}_{k,l}}{n(n-1)} + 2\frac{S^{\nu}_{k,l}}{n(n-1)} \le 1.$$

Therefore, $\langle \mu_{C_k,C_l}, \nu_{C_k,C_l} \rangle$ is an IFP. Now, we can construct the IM

$$\begin{array}{c|ccc} & C_1 & \cdots & C_m \\ \hline C_1 & \langle \mu_{C_1,C_1}, \nu_{C_1,C_1} \rangle & \cdots & \langle \mu_{C_1,C_m}, \nu_{C_1,C_m} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ C_m & \langle \mu_{C_m,C_1}, \nu_{C_m,C_1} \rangle & \cdots & \langle \mu_{C_m,C_m}, \nu_{C_m,C_m} \rangle \end{array},$$

that determines the degrees of correspondence between criteria $C_1, \ldots, C_m$.

Now, following the idea from [5], we can show the geometrical interpretation of the elements of the above IM.

Let $\alpha, \beta, \gamma, \delta, \varphi \in [0, 1]$ and

$$\alpha + \beta \leq 1,$$

$$\gamma + \delta \leq 1,$$

$$\varphi \leq \min(\alpha, \delta).$$

These numbers (thresholds) determine the criteria that are in:

- strong positive consonance – if

$$\langle \mu_{C_r,C_s}, \nu_{C_r,C_s} \rangle > \langle \alpha, \beta \rangle,$$

- positive consonance – if

$$\langle \mu_{C_r,C_s}, \nu_{C_r,C_s} \rangle \geq \langle \alpha, \beta \rangle,$$

- strong negative consonance – if

$$\langle \mu_{C_r,C_s}, \nu_{C_r,C_s} \rangle < \langle \gamma, \delta \rangle,$$

- negative consonance – if

$$\langle \mu_{C_r,C_s}, \nu_{C_r,C_s} \rangle \leq \langle \gamma, \delta \rangle,$$

- dissonance – if

$$\mu_{C_r,C_s} < \alpha, \nu_{C_r,C_s} < \delta \text{ and } \mu_{C_r,C_s} + \nu_{C_r,C_s} \geq \varphi,$$

- uncertainty – if

$$\mu_{C_r,C_s} + \nu_{C_r,C_s} < \varphi$$

(see Fig. 2).

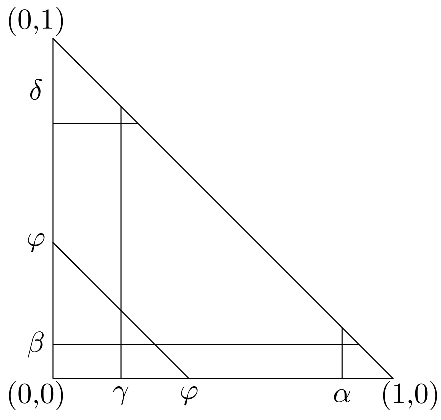

Fig. 2.

For $\alpha, \beta, \gamma, \delta$ we can use, e.g.

$$\alpha = \delta = 1 - \omega, \quad \beta = \gamma = \omega,$$

or

$$\alpha = \delta = \frac{2}{3}, \quad \beta = \gamma = \frac{1}{3},$$

or

$$\alpha = \delta = \frac{3}{4}, \quad \beta = \gamma = \frac{1}{4}.$$

5 An illustrative example

Let us have the following sequence with natural numbers: $2, 4, 7, 11, 16$ that will play the role of objects and let the role of criteria is realized by the following arithmetic functions $\varphi, \psi, \sigma, \zeta$ that are defined as follows for the natural number

$$n = \prod_{i=1}^{k} p_i^{\alpha_i},$$

where k, α_1, α_2, ..., $\alpha_k \geq 1$ are natural numbers and $p_1, p_2, ..., p_k$ are different prime numbers:

$$\begin{aligned}
\varphi(n) &= \prod_{i=1}^{k} p_i^{\alpha_i - 1}(p_i - 1), \\
\psi(n) &= \prod_{i=1}^{k} p_i^{\alpha_i - 1}(p_i + 1), \\
\sigma(n) &= \prod_{i=1}^{k} \frac{p_i^{\alpha_i + 1} - 1}{p_i - 1}, \\
\zeta(n) &= \sum_{i=1}^{k} \alpha_i . p_i .
\end{aligned}$$

By definition, for all these functions the following equalities are valid:

$$\varphi(1) = \rho(1) = \psi(1) = \sigma(1) = \delta(1) = \zeta(1) = 1.$$

n	$\varphi(n)$	$\psi(n)$	$\sigma(n)$	$\zeta(n)$
6	2	12	12	5
7	6	8	8	7
8	4	12	15	6
9	6	12	13	6
10	4	18	18	7

If we use the standard ICA, we will obtain the results:

	n	$\varphi(n)$	$\psi(n)$	$\sigma(n)$	$\zeta(n)$
n	$(1, 0)$	$(0.8, 0.1)$	$(0.8, 0.1)$	$(0.8, 0.1)$	$(0.7, 0.2)$
$\varphi(n)$	$(0.8, 0.1)$	$(1, 0)$	$(0.7, 0.2)$	$(0.7, 0.3)$	$(0.8, 0.1)$
$\psi(n)$	$(0.8, 0.1)$	$(0.6, 0.2)$	$(1, 0)$	$(0.9, 0.1)$	$(0.6, 0.2)$
$\sigma(n)$	$(0.8, 0.1)$	$(0.6, 0.2)$	$(0.9, 0.1)$	$(1, 0)$	$(0.6, 0.3)$
$\zeta(n)$	$(0.8, 0.1)$	$(0.8, 0.1)$	$(0.6, 0.2)$	$(0.6, 0.2)$	$(1, 0)$

The results of the new version of ICA are the following:

	n	$\varphi(n)$	$\psi(n)$	$\sigma(n)$	$\zeta(n)$
n	$(1.0, 0.0)$	$(0.7, 0.2)$	$(0.6, 0.1)$	$(0.6, 0.2)$	$(0.8, 0.0)$
$\varphi(n)$	$(0.7, 0.2)$	$(1.0, 0.0)$	$(0.4, 0.4)$	$(0.6, 0.1)$	$(0.7, 0.1)$
$\psi(n)$	$(0.6, 0.1)$	$(0.4, 0.4)$	$(1.0, 0.0)$	$(0.6, 0.0)$	$(0.6, 0.1)$
$\sigma(n)$	$(0.6, 0.2)$	$(0.6, 0.1)$	$(0.6, 0.0)$	$(1.0, 0.0)$	$(0.6, 0.2)$
$\zeta(n)$	$(0.8, 0.0)$	$(0.7, 0.1)$	$(0.6, 0.1)$	$(0.6, 0.2)$	$(1.0, 0.0)$

It is seen that the degrees for μ and ν are rather close but the presence of threshold values enhances the degree of uncertainty which results in the differences. That degree of uncertainty is strongly exhibited when working with natural numbers, and less exhibited when real numbers are in use.

6 Conclusion

The new version of ICA gives the possibility to establish the proximities between the evaluated objects with respect to defined threshold values. This new version, in combination with the standard ICA analysis, gives the opportunity to reveal new properties of the processed data. For instance, from the example discussed above, it is seen how the increase of n leads to changes in the numbers across the separate columns.

Acknowledgments

The authors are thankful for the support provided by the Bulgarian National Science Fund under Grant Ref. No. KP-06-N22-1/2018 "Theoretical research and applications of InterCriteria Analysis".

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