ICrAData – Software for InterCriteria Analysis

Nikolay Ikonomov

Institute of Mathematics and Informatics
Bulgarian Academy of Sciences

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InterCriteria Analysis [1] is based on Index Matrices [2] and Intuitionistic Fuzzy Sets [3].

Let an index matrix be given, where O_n are the objects, and C_n are the criteria by which the objects are evaluated:

	O_1	O_2	$\dots$	O_n
C_1	$C_1(O_1)$	$C_1(O_2)$	$\dots$	$C_1(O_n)$
C_2	$C_2(O_1)$	$C_2(O_2)$	$\dots$	$C_2(O_n)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
C_m	$C_m(O_1)$	$C_m(O_2)$	$\dots$	$C_m(O_n)$

The criteria matrix, created from the index matrix, is:

C_1	$C_1(O_1) - C_1(O_2)$	$C_1(O_1) - C_1(O_3)$ $\dots$	$C_1(O_1) - C_1(O_n)$	$C_1(O_2) - C_1(O_3)\dots$
C_2	$C_2(O_1) - C_2(O_2)$	$C_2(O_1) - C_2(O_3)$ $\dots$	$C_2(O_1) - C_2(O_n)$	$C_2(O_2) - C_2(O_3)\dots$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
C_n	$C_n(O_1) - C_n(O_2)$	$C_n(O_1) - C_n(O_3)$ $\dots$	$C_n(O_1) - C_n(O_n)$	$C_n(O_2) - C_n(O_3)\dots$

Let's demonstrate the algorithm with specific values:

	O_1	O_2	O_3	O_4	O_5
C_1	6	5	3	7	6
C_2	7	7	8	1	3
C_3	4	3	5	9	1
C_4	4	5	6	7	8

The criteria matrix is:

	(1-2)	(1-3)	(1-4)	(1-5)	(2-3)	(2-4)	(2-5)	(3-4)	(3-5)	(4-5)
C_1	1	3	-1	0	2	-2	-1	-4	-3	1
C_2	0	-1	6	4	-1	6	4	7	5	-2
C_3	1	-1	-5	3	-2	-6	2	-4	4	8
C_4	-1	-2	-3	-4	-1	-2	-3	-1	-2	-1

We create a new matrix, which takes only the sign from each value of the criteria matrix:

S_1	1	1	-1	0	1	-1	-1	-1	-1	1
S_2	0	-1	1	1	-1	1	1	1	1	-1
S_3	1	-1	-1	1	-1	-1	1	-1	1	1
S_4	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

The final matrix (which is the result) is obtained by comparing each row with all rows of the criteria matrix:

	C_1	C_2	C_3	C_4
C_1	$S_1 \# S_1$	$S_1 \# S_2$	$S_1 \# S_3$	$S_1 \# S_4$
C_2	-	$S_2 \# S_2$	$S_2 \# S_3$	$S_2 \# S_4$
C_3	-	-	$S_3 \# S_3$	$S_3 \# S_4$
C_4	-	-	-	$S_4 \# S_4$

Method μ-**biased**.

We use these comparisons for matrix μ: $0 = 0$, $1 = 1$, $-1 = -1$.
Also these comparisons for matrix ν: $-1 \neq 1$, $1 \neq -1$.

S_1	1	1	-1	0	1	-1	-1	-1	-1	1
S_2	0	-1	1	1	-1	1	1	1	1	-1
S_3	1	-1	-1	1	-1	-1	1	-1	1	1
S_4	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

μ	C_1	C_2	C_3	C_4
C_1	1	0	0.5	0.5
C_2	-	1	0.5	0.3
C_3	-	-	1	0.5
C_4	-	-	-	1

ν	C_1	C_2	C_3	C_4
C_1	0	0.8	0.4	0.4
C_2	-	0	0.4	0.6
C_3	-	-	0	0.5
C_4	-	-	-	0

We count the equal elements (for matrix μ) between two rows, and then we divide by the number of columns. Non-equal elements for matrix ν.

Method **Unbiased**.

Comparisons for matrix μ: $1 = 1$, $-1 = -1$.
Comparisons for matrix ν: $-1 \neq 1$, $1 \neq -1$.
The comparison 0 and 0 is not counted.

S_1	1	1	-1	0	1	-1	-1	-1	-1	1
S_2	0	-1	1	1	-1	1	1	1	1	-1
S_3	1	-1	-1	1	-1	-1	1	-1	1	1
S_4	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

μ	C_1	C_2	C_3	C_4
C_1	0.9	0	0.5	0.5
C_2	-	0.9	0.5	0.3
C_3	-	-	1	0.5
C_4	-	-	-	1

ν	C_1	C_2	C_3	C_4
C_1	0	0.8	0.4	0.4
C_2	-	0	0.4	0.6
C_3	-	-	0	0.5
C_4	-	-	-	0

Method ν-**biased**.

Comparisons for matrix μ: $1 = 1$, $-1 = -1$.
Comparisons for matrix ν: $0 \neq 0$, $-1 \neq 1$, $1 \neq -1$.
The comparison 0 and 0 is counted for matrix ν.

S_1	1	1	-1	0	1	-1	-1	-1	-1	1
S_2	0	-1	1	1	-1	1	1	1	1	-1
S_3	1	-1	-1	1	-1	-1	1	-1	1	1
S_4	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

μ	C_1	C_2	C_3	C_4
C_1	0.9	0	0.5	0.5
C_2	-	0.9	0.5	0.3
C_3	-	-	1	0.5
C_4	-	-	-	1

ν	C_1	C_2	C_3	C_4
C_1	0.1	0.8	0.4	0.4
C_2	-	0.1	0.4	0.6
C_3	-	-	0	0.5
C_4	-	-	-	0

Method **Balanced**.

We compute the methods μ-biased and ν-biased.
The elements for matrix μ are equal to: $(\mu_{\mu\text{-biased}} + \mu_{\nu\text{-biased}})/2$
The elements for matrix μ are equal to:$(\nu_{\mu\text{-biased}} + \nu_{\nu\text{-biased}})/2$
The comparison 0 and 0 is distributed as half of it for matrix μ and the other half for matrix ν.

S_1	1	1	-1	0	1	-1	-1	-1	-1	1
S_2	0	-1	1	1	-1	1	1	1	1	-1
S_3	1	-1	-1	1	-1	-1	1	-1	1	1
S_4	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

μ	C_1	C_2	C_3	C_4
C_1	0.95	0	0.5	0.5
C_2	-	0.95	0.5	0.3
C_3	-	-	1	0.5
C_4	-	-	-	1

ν	C_1	C_2	C_3	C_4
C_1	0.05	0.8	0.4	0.4
C_2	-	0.05	0.4	0.6
C_3	-	-	0	0.5
C_4	-	-	-	0

Method **Weighted**.

We compute the method Unbiased. Then we create new matrix P, which is the elementwise sum of the matrices μ and ν of the method Unbiased: $P = \mu_{\text{unbiased}} + \nu_{\text{unbiased}}$.

$$\mu_{\text{weighted}} := \mu_{\text{unbiased}} + \frac{\mu_{\text{unbiased}}}{P}(1 - P) = \frac{\mu_{\text{unbiased}}}{P}$$
$$\nu_{\text{weighted}} := \nu_{\text{unbiased}} + \frac{\nu_{\text{unbiased}}}{P}(1 - P) = \frac{\nu_{\text{unbiased}}}{P}$$

Of course, for $P[i][j] \neq 0$. If $P[i][j] = 0$, then we assign $1/2$ for this element for both matrices.

We recall the matrices from method Unbiased:

μ	C_1	C_2	C_3	C_4
C_1	0.9	0	0.5	0.5
C_2	-	0.9	0.5	0.3
C_3	-	-	1	0.5
C_4	-	-	-	1

ν	C_1	C_2	C_3	C_4
C_1	0	0.8	0.4	0.4
C_2	-	0	0.4	0.6
C_3	-	-	0	0.5
C_4	-	-	-	0

Matrix P:

P	C_1	C_2	C_3	C_4
C_1	0.9	0.8	0.9	0.9
C_2	-	0.9	0.9	0.9
C_3	-	-	1	1
C_4	-	-	-	1

The new matrices for method Weighted:

μ	C_1	C_2	C_3	C_4
C_1	1	0	0.5556	0.5556
C_2	-	1	0.5556	0.3333
C_3	-	-	1	0.5
C_4	-	-	-	1

ν	C_1	C_2	C_3	C_4
C_1	0	1	0.4444	0.4444
C_2	-	0	0.4444	0.6667
C_3	-	-	0	0.5
C_4	-	-	-	0

Available types for ICrA calculations

Standard ICrA – apply ICrA to a single matrix or one set of matrices.
Ordered Pair – apply ICrA to two matrices or two sets of matrices,
all three types allow ordered pair as input.

Second Order ICrA – load at least 3 matrices of the same size, compute Standard ICrA for each of them, take upper triangular matrix from μ and ν from each set, write as rows of new input matrices μ and ν, since we have two input matrices – apply Standard ICrA for ordered pair.

Aggregated ICrA – load at least 3 matrices of the same size, compute Standard ICrA for each of them, then aggregate elementwise the resulting set of matrices μ and ν: average, max-min, or min-max aggregation.

Ordered pair (μ, ν)

The ordered pair has four comparisons: greater than, less than, equal, incomparable. Let two ordered pairs (a_1, b_1) and (a_2, b_2) be given. Then:

- $(a_1, b_1) > (a_2, b_2)$ when $a_1 \geq a_2$, $b_1 < b_2$ or $a_1 > a_2$, $b_1 \leq b_2$,
- $(a_1, b_1) < (a_2, b_2)$ when $a_1 \leq a_2$, $b_1 > b_2$ or $a_1 < a_2$, $b_1 \geq b_2$,
- $(a_1, b_1) = (a_2, b_2)$ when $a_1 = a_2$, $b_1 = b_2$,
- incomparable: in the remaining cases, for example $a_1 < a_2$, $b_1 < b_2$.

Note that a_1, b_1, a_2, b_2 are integer or real values.
The standard method is a difference between each two values,
while this one is a comparison between each two pairs.

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- Atanassov K., *Index Matrices: Towards an Augmented Matrix Calculus*, Studies in Computational Intelligence, 573, 2014.
- Atanassov K., *On Intuitionistic Fuzzy Sets Theory*, Springer, Berlin, 2012.
- http://intercriteria.net/software/
- http://justmathbg.info/icradata.html

Thank you for the attention!

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