Defining Consonance Thresholds in InterCriteria Analysis: An Overview

Lyubka Doukovska, Vassia Atanassova, Evdokia Sotirova, Ivelina Vardeva and Irina Radeva

Abstract The present paper aims to provide an overview of the development of the approaches adopted in defining the consonance thresholds in the recently proposed method for decision support named InterCriteria Analysis (ICA). Discussing the rationale of this leg of the ICA research, and the motivation behind each of the subsequent steps, we trace here the gradual progress in defining the thresholds of the membership and non-membership parts of the intuitionistic fuzzy pairs serving as estimations of the pairwise consonances. This progress is based on both our deepening understanding of the ICA method, and the constant observations being made during the application of ICA to a wide range of different real-life problems and datasets.

Keywords Intercriteria analysis · Decision making
Multicriteria decision making · Intuitionistic fuzzy sets · Uncertainty · Thresholds

1 Introduction

InterCriteria Analysis (ICA) is a novel mathematical method, based on the paradigms of intuitionistic fuzzy sets and index matrices, which has been recently developed in Bulgaria with the aim to support decision making in multiobject multicriteria problems. In the originally formulated problem that gave rise to the method, a part of

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the criteria in an industrial multicriteria decision making problem exhibit high complexity and cost of the measurement compared to the other criteria. The assignment is to develop a method for identification of strong enough correlations between the cost-unfavourable criteria and the cost-favourable ones, in order to justifiably skip measurements against these cost-unfavourable criteria for at least part of the objects, and thus make the whole decision making process faster or cheaper. For the sake of terminological precision, in ICA, the term “correlation” is being replaced to the terms “positive/negative consonance” or “dissonance,” inspired from the field of cognitive maps.

As input data, the method requires an $m \times n$ table with the measurements or evaluations of m objects against n criteria. As a result, it returns an $n \times n$ table with intuitionistic fuzzy pairs, defining the degrees of consonance between each pair of criteria, hence the name “intercriteria.” Alternatively, for easier manipulation, the developed ICA software returns two $n \times n$ tables with the membership and the non-membership parts of the respective intuitionistic fuzzy pairs.

The algorithm is completely data driven and dependent on the input data from the measurements, and its present so far works well with complete datasets, without any missing values. The essence of the method is in the exhaustive pairwise comparison of the values of the measurements of all objects in the set against pairs of criteria, with all possible pairs being traversed, while counters being maintained for the percentage of the cases when the relations between the pairs of evaluations have been ‘greater than,’ ‘less than’ or ‘equal.’

The method has been proposed and described in details in 2014 [1], and extensively researched in the next two years in theoretical aspect (e.g. [2–6]), with a software application being developed (see [7, 8]). The ICA method has been extensively researched not only in the light of the originally formulated industrial problem (see [9, 10]), but also for its applicability to various multicriteria multiobjects problems (e.g. [11–14]) and with the aim of improving the performance of different procedures for mathematical optimization (e.g. [15–19]). However, ICA is still being a very new field of research, giving opportunities for discussion, comparison, approbation, validation and testing with different datasets.

2 Basic Concepts and Method

The ICA method is based on two fundamental concepts: intuitionistic fuzzy sets and index matrices. Intuitionistic fuzzy sets defined by Atanassov (see [11, 20–22]) represent an extension of the concept of fuzzy sets, as defined by Zadeh [23], exhibiting function $\mu_A(x)$ defining the membership of an element x to the set A, evaluated in the [0; 1]-interval. The difference between fuzzy sets and intuitionistic fuzzy sets (IFSs) is in the presence of a second function $\nu_A(x)$ defining the non-membership of the element x to the set A, where $\mu_A(x)$ [0; 1], $\nu_A(x)$ [0; 1], and moreover $(\mu_A(x) + \nu_A(x))$ [0; 1].

The second concept on which the proposed method relies is the concept of index matrix, a matrix which features two index sets. The theory behind the index matrices is originally described in [24] and elaborated in details in [25]. Here we will start with the index matrix M with index sets with m rows $\{C_1, \ldots, C_m\}$ and n columns $\{O_1, \ldots, O_n\}$:

$$M = \begin{array}{c|ccccccc} & O_1 & \cdots & O_k & \cdots & O_l & \cdots & O_n \\ \hline C_1 & a_{C_1,O_1} & \cdots & a_{C_1,O_k} & \cdots & a_{C_1,O_l} & \cdots & a_{C_1,O_n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ C_i & a_{C_i,O_1} & \cdots & a_{C_i,O_k} & \cdots & a_{C_i,O_l} & \cdots & a_{C_i,O_n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ C_j & a_{C_j,O_1} & \cdots & a_{C_j,O_k} & \cdots & a_{C_j,O_l} & \cdots & a_{C_j,O_n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ C_m & a_{C_m,O_1} & \cdots & a_{C_m,O_j} & \cdots & a_{C_m,O_l} & \cdots & a_{C_m,O_n} \end{array},$$

where for every p, q ($1 \le p \le m$, $1 \le q \le n$), C_p is a criterion (in our case, one of the twelve pillars), O_q in an evaluated object (in our case, one of the EU28 member states), $a_{C_p O_q}$ is the evaluation of the q-th object against the p-th criterion, and it is defined as a real number or another object that is comparable according to relation R with all the rest elements of the index matrix M, so that for each i, j, k it holds the relation $R\left(a_{C_k O_i}, a_{C_k O_j}\right)$. The relation R has dual relation $\bar{R}$, which is true in the cases when relation R is false, and vice versa.

For the needs of our method, pairwise comparisons between every two different criteria are made along all evaluated objects. During the comparison, it is maintained one counter of the number of times when the relation R holds, and another counter for the dual relation. Let $S^{\mu}_{k,l}$ be the number of cases where the relations $R\left(a_{C_k O_i}, a_{C_k O_j}\right)$ and $R\left(a_{C_l O_i}, a_{C_l O_j}\right)$ are simultaneously satisfied. Let also $S^{\nu}_{k,l}$ be the number of cases in which the relations $R\left(a_{C_k O_i}, a_{C_k O_j}\right)$ and its dual $\overline{R}\left(a_{C_l O_i}, a_{C_l O_j}\right)$ are simultaneously satisfied. As the total number of pairwise comparisons between the object is $n(n-1)/2$, it is seen that there hold the inequalities:

$$0 \le S^{\mu}_{k,l} + S^{\nu}_{k,l} \le \frac{n(n-1)}{2}.$$

For every k, l, such that $1 \le k \le l \le m$, and for $n \ge 2$ two numbers are defined:

$$\mu_{C_k,C_l} = 2\frac{S^{\mu}_{k,l}}{n(n-1)}, \quad \nu_{C_k,C_l} = 2\frac{S^{\nu}_{k,l}}{n(n-1)}.$$

The pair constructed from these two numbers plays the role of the intuitionistic fuzzy evaluation of the relations that can be established between any two criteria C_k

and C_l. In this way the index matrix M that relates evaluated objects with evaluating criteria can be transformed to another index matrix $M*$ that gives the relations among the criteria:

$$M^* = \begin{array}{c|ccc} & C_1 & \ldots & C_m \\ \hline C_1 & \langle \mu_{C_1,C_1}, \nu_{C_1,C_1} \rangle & \ldots & \langle \mu_{C_1,C_m}, \nu_{C_1,C_m} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ C_m & \langle \mu_{C_m,C_1}, \nu_{C_1,C_m} \rangle & \ldots & \langle \mu_{C_m,C_m}, \nu_{C_m,C_m} \rangle \end{array}$$

The final step of the algorithm is to determine the degrees of correlation between the criteria, depending on the user's choice of μ and ν. We call these correlations between the criteria: 'positive consonance,' 'negative consonance' or 'dissonance.'

Let α, β [0; 1] be given, so that $\alpha + \beta \leq 1$. We say that criteria C_k and C_l are in:

- (α, β)-positive consonance, if $\mu_{C_k,C_l} > \alpha$ and $\nu_{C_k,C_l} < \beta$;
- (α, β)-negative consonance, if $\mu_{C_k,C_l} < \beta$ and $\nu_{C_k,C_l} > \alpha$;
- (α, β)-dissonance, otherwise, [1].

Obviously, the larger α and/or the smaller β, the less number of criteria may be simultaneously connected with the relation of (α, β)-positive consonance. For practical purposes, it carries the most information when either the positive or the negative consonance is as large as possible, while the cases of dissonance are less to no informative.

3 Defining the ICA Thresholds: Overview

According to the authors, one of the most important aspects of the method is the way the thresholds are determined: as this is what gives the measure of precision of the decision made using ICA. Setting the thresholds can, obviously, be a completely expert's decision, but an algorithmic approach is considered to yield more precision, objectivity and sustainability. This is why a series of papers has explored this issue in a stepwise manner, based on both our deepening understanding of the ICA method, and observations made during the application of the method to various real-life datasets. It is noteworthy that from the very first contemplations of the method, we realized that although both the values of the intercriteria consonance, and respectively the thresholds against these being measured, are normed within the [0, 1]-interval, the thresholds are not universal, and different thresholds may be adequate in different problem areas of application. In other words, there is no "one size fits all" solution when defining the ICA thresholds and it is worth specifically exploring how ICA membership and non-membership thresholds α and β can be objectively defined.

Historically, "*the immediate first idea*" (per [26]) of this leg of research involved setting predefined values, i.e. numbers from the [0, 1]-interval, relatively high for

Table 1 Results from application of ICA over a dataset of EU28 member states' competitiveness in year 2013–2014, based on the data from [28]

μ	1	2	3	4	5	6	7	8	9	10	11	12
1	1.000	0.735	0.577	0.720	0.807	0.836	0.733	0.749	0.854	0.503	0.804	0.844
2	0.735	1.000	0.479	0.661	0.749	0.677	0.537	0.590	0.786	0.661	0.804	0.799
3	0.577	0.479	1.000	0.421	0.519	0.558	0.627	0.675	0.550	0.413	0.548	0.556
4	0.720	0.661	0.421	1.000	0.730	0.683	0.590	0.563	0.677	0.497	0.712	0.690
5	0.807	0.749	0.519	0.730	1.000	0.735	0.622	0.632	0.775	0.579	0.815	0.847
6	0.836	0.677	0.558	0.683	0.735	1.000	0.749	0.712	0.788	0.466	0.759	0.751
7	0.733	0.537	0.627	0.590	0.622	0.749	1.000	0.741	0.685	0.399	0.624	0.624
8	0.749	0.590	0.675	0.563	0.632	0.712	0.741	1.000	0.712	0.497	0.688	0.680
9	0.854	0.786	0.550	0.677	0.775	0.788	0.685	0.712	1.000	0.526	0.810	0.831
10	0.503	0.661	0.413	0.497	0.579	0.466	0.399	0.497	0.526	1.000	0.611	0.598
11	0.804	0.804	0.548	0.712	0.815	0.759	0.624	0.688	0.810	0.611	1.000	0.873
12	0.844	0.799	0.556	0.690	0.847	0.751	0.624	0.680	0.831	0.598	0.873	1.000

(a) Membership parts of the IF pairs

ν	1	2	3	4	5	6	7	8	9	10	11	12
1	0.000	0.220	0.386	0.188	0.132	0.077	0.185	0.172	0.090	0.452	0.138	0.111
2	0.220	0.000	0.466	0.228	0.172	0.228	0.362	0.317	0.146	0.286	0.135	0.138
3	0.386	0.466	0.000	0.476	0.405	0.344	0.286	0.251	0.394	0.537	0.394	0.389
4	0.188	0.228	0.476	0.000	0.143	0.169	0.283	0.307	0.201	0.397	0.175	0.198
5	0.132	0.172	0.405	0.143	0.000	0.153	0.272	0.259	0.135	0.341	0.098	0.079
6	0.077	0.228	0.344	0.169	0.153	0.000	0.135	0.169	0.101	0.439	0.143	0.159
7	0.185	0.362	0.286	0.283	0.272	0.135	0.000	0.146	0.209	0.505	0.267	0.275
8	0.172	0.317	0.251	0.307	0.259	0.169	0.146	0.000	0.206	0.415	0.217	0.233
9	0.090	0.146	0.394	0.201	0.135	0.101	0.209	0.206	0.000	0.405	0.119	0.101
10	0.452	0.286	0.537	0.397	0.341	0.439	0.505	0.415	0.405	0.000	0.328	0.344
11	0.138	0.135	0.394	0.175	0.098	0.143	0.267	0.217	0.119	0.328	0.000	0.071
12	0.111	0.138	0.389	0.198	0.079	0.159	0.275	0.233	0.101	0.344	0.071	0.000

(b) Non-membership parts of the IF pairs

the membership threshold, like numbers in the interval [0.75, 1], and relatively low for the non-membership threshold β like numbers in the interval [0, 0.25]. The research (published in paper [27]) started from the ideal membership threshold of 1 and non-membership threshold of 0, and involved gradual decrease ofα and increase of, checking on each step what new intercriteria pairs "emerge." Checking the ICA pairs against both thresholds was consequent, with first checking the emergent ICA pairs when applying the membership threshold, and then the non-membership one.

In [27], we presented the essence of the idea, applying the method to data from the Global Competitiveness Report of the World Economic Forum [28], where we were interested to detect the eventual correlations between the 12 'pillars of country competitiveness,' in order to outline those fewer pillars on which policy makers should concentrate their efforts. Our motivation to conduct the analysis has been that it might be expected that improved country's performance against some pillars would positively affect the country's performance in the respective correlating ones. This is in line with WEF's address to state policy makers to "*identify and strengthen the transformative forces that will drive future economic growth*" of the countries, as formulated in the Preface of the GCR in the year 2013–2014 [28].

The results from application of the ICA method over the data from this dataset is given in the two tables below (Table 1a, b), where for each pair of criteria C_i – C_j the degree of correlation (consonance) is given by the intuitionistic fuzzy (IF) pair $\langle \mu_{C_i,C_j}, \nu_{C_i,C_j} \rangle$. For every criterion, the degree of correlation $C_i - C_i$ is $\langle 1, 0 \rangle$, because every criterion perfectly correlates only with itself; also the tables are symmetrical along the main diagonal, because the degree of correlation of C_i – C_j equals that of C_j – C_i.

In [27], we showed how, given a fixed year (i.e. dataset), we variate the thresholds α, β, which respectively changes the number of criteria that start correlating, hence, the positive consonances formed between the pairs of criteria. As an illustration, we

Table 2 List of positive intercriteria consonances per different pairs of thresholds

Values for (α; β)	List of positive consonances (PC)	No. of PC	No. of criteria
(0.85; 0.15)	**1–9; 11–12**	2	4
	1–5; 1–6; 1–9; **1–11; 1–12; 2–11; 5–11; 5–12; 9–11; 9–12;** 11–12	11	7
(0.75; 0.25)	1–5; 1–6; 1–9; 1–11; 1–12; **2–9;** 2–11; **2–12; 5–9;** 5–11; 5–12; **6–9; 6–11; 6–12;** 9–11; 9–12; 11–12	17	7
(0.70; 0.30)	**1–2; 1–4;** 1–5; 1–6; **1–7; 1–8;** 1–9; 1–11; 1–12; **2–5;** 2–9; 2–11; 2–12; **4–5; 4–11; 5–6;** 5–9; 5–11; 5–12; **6–7; 6–8;** 6–9; 6–11; 6–12; **7–8; 8–9;** 9–11; 9–12; 11–12	29	10
(0.65; 0.35)	1–2; 1–4; 1–5; 1–6; 1–7; 1–8; 1–9; 1–11; 1–12; **2–4;** 2–5; **2–6;** 2–9; **2–10;** 2–11; 2–12; **3–8;** 4–5; **4–6; 4–9;** 4–11; **4–12;** 5–6; 5–9; 5–11; 5–12; 6–7; 6–8; 6–9; 6–11; 6–12; 7–8; **7–9;** 8–9; **8–11; 8–12;** 9–11; 9–12; 11–12	39	12

give here (Table 2) the results from the check of the dataset of EU28 countries from the GCR in the year 2013–2014 how stepwise changing the thresholds α, β with step of 0.05 leads to "emergence" of new intercriteria positive consonances and new correlating criteria (those highlighted with bold typeface).

In the paper [27], a finer step, with which the thresholds α, β are changed, was taken, namely, 0.025 (decrease for α, increase for β) starting with (0.85, 0.15) and ending with (0.7, 0.3). The observations of the results lead us to the conclusions that in the low end, under a certain value for threshold α (respectively, above a certain value for threshold β), it is natural that *all* criteria start exhibiting some (rather low) degree of consonance, which is ineffective for the analysis. In the high end, with too high threshold α, respectively too threshold β, so few, if any, criteria are in positive consonance that this is also not effective either. Thus, specifically for the case of the global competitiveness of the EU28 countries, we concluded to focus the analysis of intercriteria consonances in the thresholds range from (0.775; 0.225) to (0.825; 0.175), with some more specific discussions about the findings being made.

In [29], already it was noted that this approach may yield some rather different results for α and β. It was commented that paper [27] adopted the "*simplistic case*," where the values of the thresholds α and β in the different pairs of numbers were always summing up to 1, like (0.85; 0.15), (0.825; 0.175), (0.8; 0.2), etc. This however helped us could notice that with this setting applied over the data in the dataset, threshold β produces disproportionately many intercriteria pairs, far beyond the number of pairs produced by the respective threshold α. As we summarized in Table 4 in [30], when $\alpha = 0.85$, the outlined pairs in positive consonance are only 2, and when $\beta = 1 - \alpha = 0.15$, the outlined pairs in negative consonance are 19. When $(\alpha, \beta) = (0.80; 0.20)$, these numbers are respectively 11 and 29; and so forth, using

Table 3 Finding maximal correlations per criterion

Column (1)	Column (2)
C_1	$\max\limits_{j, j \neq 1} \mu(C_1, C_j)$
C_2	$\max\limits_{j, j \neq 2} \mu(C_2, C_j)$
…	…
C_n	$\max\limits_{j, j \neq n} \mu(C_n, C_j)$

a step of 0.05, until we reach the intuitionistic fuzzy pair of thresholds (0.65; 0.35), when these numbers are respectively 39 and 51.

This led to the idea of a finer approach in which one of the thresholds, interchangeably, was fixed, and the other one was to be accordingly determined, in order to yield commensurate (if not identical) results. This required redefinition of the problem of determining the thresholds: it was no more the question to determine the exact values of the thresholds α and β, and thus obtain which pairs of criteria exhibit high positive consonance, but it was the reverse problem: in order to obtain the values of both thresholds it was necessary to determine a number $0 < k < n$ and find which are those k (out of all n) criteria, which exhibit the highest positive intercriteria consonances.

For this purpose, in [26] a simple algorithm was developed, which for every separate criterion calculates its positive consonance with each of the rest $(n - 1)$ criteria, and takes the maximal value; and then sorts these n values in a descending way, selecting the top k of them. However, it was noted that following such an algorithm it is quite possible to "draw the line" at a wrong place: the differences between the α-s and β-s of the first $(k + 1)$ criteria and between the α-s and β-s of the first k criteria might be negligible, while the differences between the α-s and β-s of the first $(k - 1)$ criteria and between the α-s and β-s of the first k criteria might be quite well expressed. Later, this motivated the discussion in [31], aimed to prescind from a particular number k, but attempt to identify the (unknown in advance) "most strongly correlating criteria."

The algorithm, developed in [26] is the following.

(1) For each criterion C_i, $i = 1, \ldots, n$, we find $\max\limits_{j, j \neq i} \mu(C_i, C_j)$, i.e. the maximum of the discovered correlations of C_i with all the rest criteria C_j, $j = 1, \ldots, n$, $i \neq j$. Thus we obtain for each criterion, which is its top-correlating value.
(2) We create a table like the one shown on Table 3.
(3) We sort the whole Table 3 by Column (2) in descending order, thus ordering the top-correlating values for all individual criteria.
(4) We shortlist the first k criteria from Column (1) in the resultant sorted table.
(5) The sought value of the threshold α is then the respective value in Column (2) on k-th place top down, in the resultant sorted table.

The algorithm is shown with the following example, which also illustrates why "mechanically" determining the number of k criteria, we would like to work with, is not obligatory producing optimal results: the gap between the value of α, needed to shortlist the top 4 correlating criteria (0.854) and α, needed to shortlist the top 5 correlating criteria (0.847) is much smaller compared to the gap between the value α, needed to shortlist the top 5 and that, needed to shortlist the top 6 correlating criteria (0.804), or the gap between the value α, needed to shortlist the top 3 and that, needed to shortlist the top 4 correlating criteria (0.873) (Fig. 1).

After sorting the column by $\max_{j, j \neq i} \mu(C_i, C_j)$ in descending order, we obtain the table from Fig. 2.

This concluded the algorithm for determining the membership threshold α. In order to determine the respective value for the non-membership threshold β, we repeat the algorithm for β in a mirror-like way: we find $\min_{j, j \neq i} \nu(C_i, C_j)$, i.e. the minimums of the discovered correlations of C_i with all the rest criteria C_j, $j = 1, \ldots, n$, $i \neq j$, and then sort them in ascending order. Then we take the *minimal superset of criteria* ordered by negative consonance, which contains the set of criteria as defined by the number k in the algorithm for defining threshold α, as graphically shown on Fig. 3.

In our example, the set of criteria ordered by positive consonance with $k = 4$ was the set $\{11, 12, 1, 9\}$ when $\alpha = 0.854$, hence the resultant minimal subset from the set of criteria ordered by positive consonance is $\{11, 12, 1, 6, 5, 9\}$ when $\beta = 0.09$. We can note that in this way the sum of α and β is no more 1. In comparison with the former version of the method of determining the thresholds by taking predefined constants summing up to 1, this would mean that if β was set to $1 - 0.854 = 0.146$, this would mean effectively that from the non-membership side 10 (rather than 6) out

μ	1	2	3	4	5	6	7	8	9	10	11	12
1		0.735	0.577	0.720	0.807	0.836	0.733	0.749	0.854	0.503	0.804	0.844
2	0.735		0.479	0.661	0.749	0.677	0.537	0.590	0.786	0.661	0.804	0.799
3	0.577	0.479		0.421	0.519	0.558	0.627	0.675	0.550	0.413	0.548	0.556
4	0.720	0.661	0.421		0.730	0.683	0.590	0.563	0.677	0.497	0.712	0.690
5	0.807	0.749	0.519	0.730		0.735	0.622	0.632	0.775	0.579	0.815	0.847
6	0.836	0.677	0.558	0.683	0.735		0.749	0.712	0.788	0.466	0.759	0.751
7	0.733	0.537	0.627	0.590	0.622	0.749		0.741	0.685	0.399	0.624	0.624
8	0.749	0.590	0.675	0.563	0.632	0.712	0.741		0.712	0.497	0.688	0.680
9	0.854	0.786	0.550	0.677	0.775	0.788	0.685	0.712		0.526	0.810	0.831
10	0.503	0.661	0.413	0.497	0.579	0.466	0.399	0.497	0.526		0.611	0.598
11	0.804	0.804	0.548	0.712	0.815	0.759	0.624	0.688	0.810	0.611		0.873
12	0.844	0.799	0.556	0.690	0.847	0.751	0.624	0.680	0.831	0.598	0.873	

C_i	$\max_{j,j \neq i} \mu(C_i, C_j)$
1	0.854
2	0.804
3	0.675
4	0.730
5	0.847
6	0.836
7	0.749
8	0.749
9	0.854
10	0.661
11	0.873
12	0.873

Fig. 1 Finding maximal correlations per criterion (Step 2 of the algorithm)

C_i	$\max_{j,j\neq i} \mu(C_i, C_j)$
11	0.873
12	0.873
1	0.854
9	0.854
5	0.847
6	0.836
2	0.804
7	0.749
8	0.749
4	0.73
3	0.675
10	0.661

Number of correlating criteria	Number of pairs of correlating criteria	Criteria ordered by positive consonance	True when $\alpha \geq$
2	1	11	0.873
		12	
4	2	1	0.854
		9	
5	3	5	0.847
6	11	2	0.804
7	13	6	0.788
9	20	7	0.749
		8	
10	25	4	0.730
11	37	3	0.675
12	39	10	0.661

$k = 4$

Fig. 2 Shortlist the first k criteria (Step 4 of the algorithm)

Number of correlating criteria	Number of pairs of correlating criteria	Criteria ordered by positive consonance	True when $\alpha \geq$
2	1	11	0.873
		12	
4	2	1	0.854
		9	
5	3	5	0.847
6	11	2	0.804
7	13	6	0.788
9	20	7	0.749
		8	
10	25	4	0.730
11	37	3	0.675
12	39	10	0.661

$k = 4$

Number of correlating criteria	Number of pairs of correlating criteria	Criteria ordered by negative consonance	True when $\beta \leq$
2	1	11	0.071
		12	
4	2	1	0.077
		6	
5	3	5	0.079
6	4	9	0.09
8	13	2	0.135
		7	
9	17	4	0.143
10	19	8	0.146
11	38	3	0.251
12	45	10	0.286

Fig. 3 Determining the appropriate threshold β with respect to threshold α

of the 12 criteria would be interacting within this threshold, which is much bigger "noise."

This "noise" resulting from the consecutive processing of the membership and non-membership thresholds was observed and solved in the next publication [32] with a solution inspired by the triangular geometric interpretation of intuitionistic fuzzy sets. The intuitionistic fuzzy triangle (proposed for the first time in [33], also see [11]) is a graphic interpretation of intuitionistic fuzzy sets that has no analogue in ordinary fuzzy sets. Its vertices (1, 0), (0, 1) and (0, 0) represent respectively the absolute Truth, the absolute Falsity and the absolute Uncertainty, while the hypotenuse is a projection of the intuitionistic fuzzy onto the fuzzy.

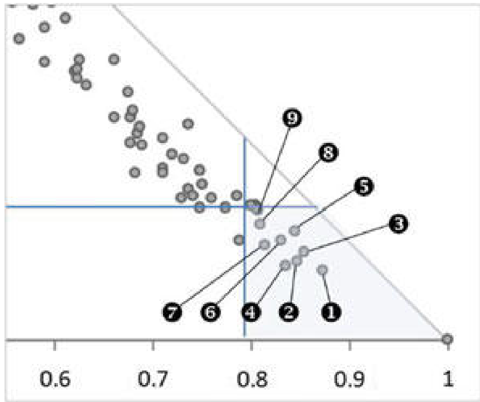


Fig. 4 Close-up of shortlisted intercriteria pairs in the IF triangle cut-out

Plotting the intercriteria pairs as points onto the intuitionistic fuzzy interpretational triangle helped for the first time to develop a procedure for short-listing the set of top consonance pairs of criteria according to both thresholds α and β simultaneously. For this purpose, for each point its distance from the (1, 0) point is calculated, where (1, 0) in the context of ICA represents the case of perfect positive consonance between two criteria (including the perfect positive consonance of any criterion with itself).

The formula for the distance d_{C_i,C_j} of the intercriteria pair (C_i, C_j) to the (1; 0) point is

$$d_{C_i,C_j} = \sqrt{\left(1 - \mu_{C_i,C_j}\right)^2 + \nu_{C_i,C_j}^2}$$

and the pairs are ordered according to their d_{C_i,C_j} sorted in ascending way. Plotting the intercriteria correlations as points in the intuitionistic fuzzy triangle gives us the possibility to rank and work with the strongest pairs of criteria, simultaneously handling membership and non-membership at a time, while in the hitherto steps of our research we have ranked and worked with the individual criteria, ordered according to one of the components in the pair, most often the membership component. In addition, this innovative approach to the theory of ICA led to a new feature in the software for calculation of intercriteria correlations, enabling their embedded graphical visualization [8].

To demonstrate the advantage of this approach, we gave in [26] the same data for the competitiveness of EU28 economies from the World Economic Forum's Global Competitiveness Report for the year 2013–2014. For convenience, the ICA software arranges the output of intercriteria consonances in two tables, one giving the membership parts (Table 4) and the other giving the non-membership parts (Table 5) of the intuitionistic fuzzy pairs for each pair of criteria. Then we give the graphic interpretation (Fig. 4) of the so-produced intuitionistic fuzzy set of points (standing for the intercriteria pairs, 66 in number for the case of 12 criteria). We provide also (the beginning of) the table (Table 6) with ordered correlating pairs, with respect to the distance from the (1, 0) point.

To define the top correlating criteria, as explained in [26], two randomly taken threshold values (namely, $\alpha = 0.796$ and $\beta = 0.134$) are taken, which yield 9 points

Table 4 Discovered membership values for the year 2013–2014

M^{μ}	1	2	3	4	5	6	7	8	9	10	11	12
1		0.74	0.58	0.72	0.81	0.84	0.73	0.75	0.85	0.5	0.8	0.84
2	0.74		0.48	0.66	0.75	0.68	0.54	0.59	0.79	0.66	0.8	0.8
3	0.58	0.48		0.42	0.52	0.56	0.63	0.67	0.55	0.41	0.55	0.56
4	0.72	0.66	0.42		0.73	0.68	0.59	0.56	0.68	0.5	0.71	0.69
5	0.81	0.75	0.52	0.73		0.74	0.62	0.63	0.78	0.58	0.81	0.85
6	0.84	0.68	0.56	0.68	0.74		0.75	0.71	0.79	0.47	0.76	0.75
7	0.73	0.54	0.63	0.59	0.62	0.75		0.74	0.69	0.4	0.62	0.62
8	0.75	0.59	0.67	0.56	0.63	0.71	0.74		0.71	0.5	0.69	0.68
9	0.85	0.79	0.55	0.68	0.78	0.79	0.69	0.71		0.53	0.81	0.83
10	0.5	0.66	0.41	0.5	0.58	0.47	0.4	0.5	0.53		0.61	0.6
11	0.8	0.8	0.55	0.71	0.81	0.76	0.62	0.69	0.81	0.61		0.87
12	0.84	0.8	0.56	0.69	0.85	0.75	0.62	0.68	0.83	0.6	0.87	

Table 5 Discovered non-membership values for the year 2013–2014

M^{ν}	1	2	3	4	5	6	7	8	9	10	11	12
1		0.22	0.39	0.19	0.13	0.08	0.19	0.17	0.09	0.45	0.14	0.11
2	0.22		0.47	0.23	0.17	0.23	0.36	0.32	0.15	0.29	0.13	0.14
3	0.39	0.47		0.48	0.4	0.34	0.29	0.25	0.39	0.54	0.39	0.39
4	0.19	0.23	0.48		0.14	0.17	0.28	0.31	0.2	0.4	0.17	0.2
5	0.13	0.17	0.4	0.14		0.15	0.27	0.26	0.13	0.34	0.10	0.08
6	0.08	0.23	0.34	0.17	0.15		0.13	0.17	0.1	0.44	0.14	0.16
7	0.19	0.36	0.29	0.28	0.27	0.13		0.15	0.21	0.51	0.27	0.28
8	0.17	0.32	0.25	0.31	0.26	0.17	0.15		0.21	0.42	0.22	0.23
9	0.09	0.15	0.39	0.2	0.13	0.1	0.21	0.21		0.4	0.12	0.10
10	0.45	0.29	0.54	0.4	0.34	0.44	0.51	0.42	0.4		0.33	0.34
11	0.14	0.13	0.39	0.17	0.10	0.14	0.27	0.22	0.12	0.33		0.07
12	0.11	0.14	0.39	0.2	0.08	0.16	0.28	0.23	0.10	0.34	0.07	

in the cut-out, i.e. 9 intercriteria positive consonance pairs, formed among a set of 6 individual criteria. As we will see, there are also other alternative way of forming the subset of top correlating intercriteria pairs.

In a consequent step of this leg of research, we explored the question of traversing and ranking elements of an intuitionistic fuzzy set in the intuitionistic fuzzy interpretation triangle per their proximity to the point (1, 0). The procedure proposed in [34] required that these threshold values α, β are known in advance and predefined, which for various reasons may not always be the case. For this purpose, the procedure requires application of the topological operators *Closure* and *Interior*, which are defined using the following formulas (see [1, 22, 35]) and illustrated on Fig. 5:

Table 6 Ordering of the correlating pairs, with respect to the distance from (1, 0) of the points that represent them in the IF interpretational triangle

No.	d_{C_i,C_j}	Criteria in (α, β)-positive consonance	μ_{C_i,C_j}	ν_{C_i,C_j}
❶	0.148	**11–12** Business sophistication—Innovation	0.87	0.07
❷	0.170	**5–12** Higher education and training—Innovation	0.85	0.08
❸	0.175	**1–9** Institutions—Technological readiness	0.85	0.09
❹	0.179	**1–6** Institutions—Goods market efficiency	0.84	0.08
❺	0.194	**1–12** Institutions—Innovation	0.84	0.11
❻	0.197	**9–12** Technological readiness—Innovation	0.83	0.10
❼	0.206	**5–11** Higher education and training—Business sophistication	0.82	0.10
❽	0.225	**9–11** Technological readiness—Business sophistication	0.81	0.12
❾	0.230	**1–5** Institutions—Higher education and training	0.81	0.13

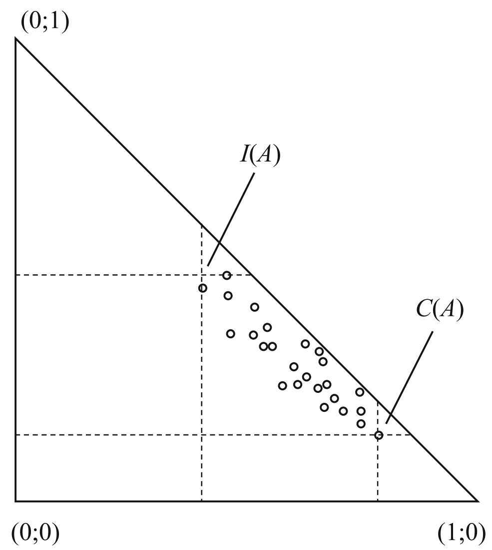


Fig. 5 An IFS, plotted onto the intuitionistic fuzzy triangle, with the indicated places of the topological operators Closure and Interior

$$C(A) = \left\{ \left\langle x, \sup_{y \in E} \mu_A(y), \inf_{y \in E} \nu_A(y) \right\rangle | x \in E \right\},$$

$$I(A) = \left\{ \left\langle x, \inf_{y \in E} \mu_A(y), \sup_{y \in E} \nu_A(y) \right\rangle | x \in E \right\}.$$

We will note that since, in the context of ICA, we are only working with finite sets of m objects, of n criteria, and therefore with a resultant finite set of $n(n-1)/2$

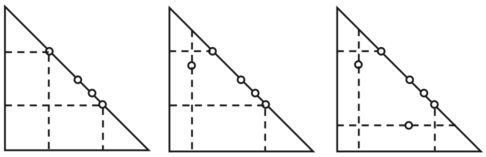
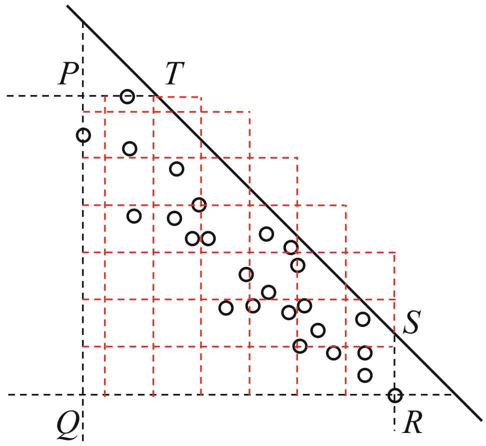


Fig. 6 Triangle, trapezium or pentagon are the possible shapes of the zone, enclosed by the topological operators Closure and Interior, and the hypotenuse of the intuitionistic fuzzy triangle

Fig. 7 The segment from the intuitionistic fuzzy triangle, containing the pentagonal zone, enclosed by the topological operators Closure and Interior (respectively, points *R* and *P*) and the hypotenuse, gridded with unit rectangles



intercriteria pairs, we can safely replace the functions 'supremum' and 'infimum,' respectively by the functions 'maximum' and 'minimum.'

We will also note that depending on the particular set, it may have the form of a triangle in the case of a fuzzy set, all of which elements are plotted onto the hypotenuse (the least challenging case), or a trapezium, or a pentagon (the most challenging case), see Fig. 6. In our research in [34] and here we discuss the most general case of a pentagon (Fig. 7).

The procedure for ranking the intercriteria pairs, hence, comprises two phases: (1) To define the unit lengths of the rectangular grid that will divide the pentagon; (2) To define the consequence of traversing the so-defined subrectangles of the grid. For completeness of the present discussion, we will note that the starting point of the so-constructed grid depends on the specific problem formulation: if the problem requires us to seek highest possible consonances among the intercriteria pairs, i.e. those closest to the Truth, represented by point (1, 0), the so-constructed grid is to start from the point (mapping) of the operator Closure (which for the needs of the example on Fig. 7 happens to be point *R*).

A possible way to define the unit lengths a, b of the rectangular grid is given with the following two formulas:

$$a = \frac{\max\limits_{y \in E} \mu_A(y) - \min\limits_{y \in E} \mu_A(y)}{\frac{n(n-1)}{2}}, \quad b = \frac{\max\limits_{y \in E} \nu_A(y) - \min\limits_{y \in E} \nu_A(y)}{\frac{n(n-1)}{2}}$$

The lengths *PQ* and *QR* are divided by the total number of points in the plotted set, and this is the finest possible division for the grid.

```
Rank 1.1.   (mu,    nu)
Rank 1.2.   (mu,    nu+1)
...
Rank 1.s.   (mu,    nu+s)

Rank 2.1.   (mu-1, nu)
Rank 2.2.   (mu-1, nu+1)
...
Rank 2.s.   (mu-1, nu+s)
Rank 2.s+1. (mu-1, nu+s+1)
...
Rank 3.1.   (mu-2, nu)
...
```

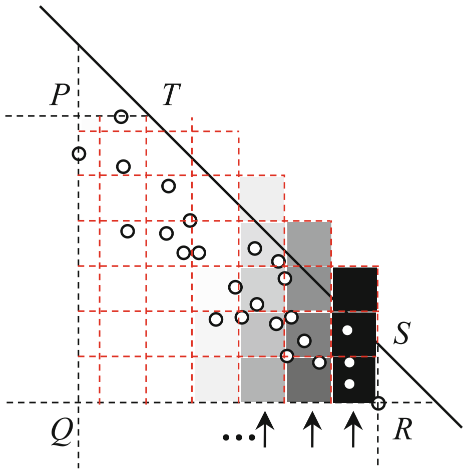


Fig. 8 Traversing the grid by the strategy "max μ first"

Another approach is to assign to a and b, respectively, the smallest possible positive, non-null difference in the first coordinates of any two points of the set, and the smallest possible positive, non-null difference in the second coordinates of any two points in the set, by the formulas:

$$a = \min_{i,j \in A}\left(\left|\mu_i - \mu_j\right|\right), \ b = \min_{k,l \in A}(|\nu_k - \nu_l|).$$

For the sake of completeness, we can also note the most obvious way of defining the unit lengths of the rectangular grid by dividing PQ and QR into predefined number(s), not necessarily the same number of sections per side. Then, for two predefined numbers u, w, the formulas will have the following forms:

$$a = \frac{\max_{y \in E} \mu_A(y) - \min_{y \in E} \mu_A(y)}{u}, b = \frac{\max_{y \in E} \nu_A(y) - \min_{y \in E} \nu_A(y)}{w}.$$

Here we make the assumption that all different intercriteria pairs whose mappings (points) that belong to a single subrectangle of the grid (if more than one) will be treated equally. This transforms the question to comparing and ranking the subrectangles of the grid, i.e., how the grid is being traversed, which essentially reduces to the question how we treat—and prioritize between—the three intuitionistic fuzzy components: membership, non-membership and uncertainty. In this relation, in [34] we proposed three different strategies, or scenarios.

(1) *Strategy "max μ first."* In response to this strategy, we start with the subrectangle with the maximal membership and minimal non-membership, i.e. the one containing the set's closure, and traverse through the grid in vertical direction (bottom-to-top), in a way that preserves the membership part as high as possible, while running through the gradually increasing non-membership parts, as illustrated in Fig. 8. The following pseudocode gives it in a more formal way:

```
Rank 1.1.   (mu,       nu)
Rank 1.2.   (mu-1,     nu)
...
Rank 1.s.   (mu-s-1,  nu)
Rank 2.1.   (mu,       nu+1)
Rank 2.2.   (mu-1,     nu+1)
...
Rank 2.s.   (mu-s-1,  nu+1)
...
Rank t.1.   (mu-1,     nu+t)
Rank t.2.   (mu-2,     nu+t)
...
Rank t.s-1. (mu-s-1,  nu+t)
...
```

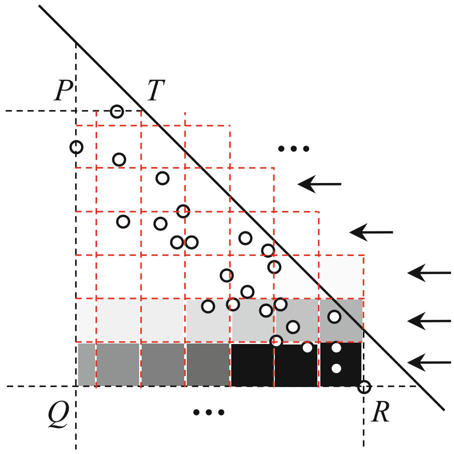


Fig. 9 Traversing the grid by the strategy "min ν first"

```
Rank 1.   (mu, nu)
Rank 2.   (mu-1, nu),
          (mu+1, nu)
Rank 3.   (mu-2, nu),
          (mu-1, nu+1),
          (mu, nu+2)
...
Rank r+1. (mu-r, nu),
          (mu-r+1, nu+1),
          (mu-r+2, nu+2),
          ...,
          (mu-2, nu+r-2),
          (mu-1, nu+r-1),
          (mu, nu+r)
...
```

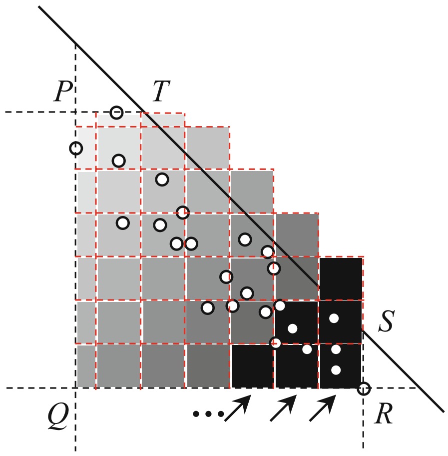


Fig. 10 Traversing the grid by the diagonal strategy

(2) *Strategy "min ν first"*. In response to this strategy, we again start with the subrectangle with the maximal membership and minimal non-membership, but this time we traverse through the grid in horizontal direction (right-to-left), in a way that preserves the non-membership part as low as possible, while running through the gradually decreasing membership parts. The illustration of this strategy follows by analogy, Fig. 9. The following pseudocode gives it in a more formal way:

(3) *Diagonal strategy.* Starting with the subrectangle with the maximal membership and minimal non-membership, we then take the simultaneously the union of the subrectangles that are located one up and one left of the previous, and so forth, as illustrated in Fig. 4 and by the following pseudocode (Fig. 10).

Any of the strategies for defining the rectangular grid can be combined with any strategy for traversing the grid; other alternatives are also possible.

In the latest development of the study of ICA thresholds determination, other alternatives were proposed about how to form the subset of top correlating intercriteria pairs. In [36], three alternatives were listed as ways to construct that subset,

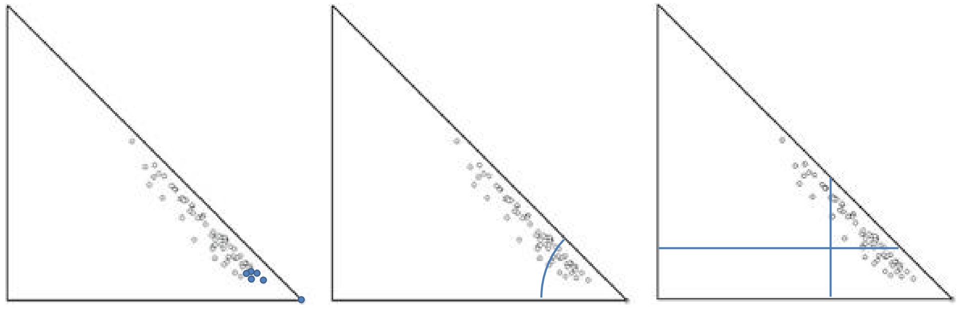


Fig. 11 Illustrating the three alternatives for constructing the subset of ICA pairs

depending on user's preference or external requirement, and a speculation was made that others are also possible. There three proposed alternatives were graphically represented as in Fig. 11, and defined as follows:

(1) Select top p or top q% of the $n(n-1)/2$ ICA pairs;
(2) Select all ICA pairs whose corresponding points are within a given radius r from the (1; 0) point;
(3) Select all ICA pairs whose corresponding points fall within the trapezoid formed between the abscissa, the hypotenuse and the two lines corresponding to $y = \alpha$, $x = \beta$ for two predefined numbers $\alpha, \in [0; 1]$ (i.e., the approach adopted earlier in [26]).

Most notably, this leg of the research in [36, 37] was dedicated to idea of having *triples* of criteria in positive consonance, upgrading the original concept for discovery of consonances between intercriteria pairs. The formulated work hypothesis was that, given a record of intercriteria pairs that have exhibited positive consonance over a longer period of time, triples and n-tuples of more criteria could be detected among them featuring high enough pairwise consonance, and an algorithm was proposed for identification and ranking of the intercriteria triples. The particular interpretation of such triple of intercriteria consonances, as well as the practical usefulness of the new concept, was discussed to be a matter of further investigation by problem-specific experts.

4 Conclusion

In the present paper, we aimed to make a detailed and justified overview of the stages of development of the research of defining the thresholds in intercriteria analysis. Starting with the definition and the first intuitive steps related to determining the thresholds, we trace its progress over time based on our deepening understanding of the ICA method and following the practical applications of ICA over diverse problems and datasets. We consider this leg of the ICA research to be one of the most important theoretical aspects, since it is the concluding step of ICA, determining the measure of precision which ICA-based decisions can be made with.

Nevertheless, we consider it appropriate other case studies and problems, approached in future with the ICA approach, all proposed algorithms—from the present and previous researches—are worth approbating, in order to compare the results for various fields of application, consult them with experts in the respective areas, and make a better justification of our choice of method of selecting threshold values and selecting the top-correlating criteria.

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References

1. K. Atanassov, D. Mavrov, V. Atanassova. Intercriteria Decision Making: A New Approach for Multicriteria Decision Making, Based on Index Matrices and Intuitionistic Fuzzy Sets. Issues in Intuitionistic Fuzzy Sets and Generalized Nets, Vol. 11, 2014, 1–8.
2. K. Atanassov, V. Atanassova, G. Gluhchev, InterCriteria Analysis: Ideas and problems, Notes on Intuitionistic Fuzzy Sets, Vol. 21, 2015, No. 1, 81–88.
3. L. Todorova, P. Vassilev, J. Surchev. Using Phi Coefficient to Interpret Results Obtained by InterCriteria Analysis, In: Novel Developments in Uncertainty Representation and Processing, Vol. 401, Advances in Intelligent Systems and Computing, Springer, 2016, 231–239.
4. N. Angelova, K. Atanassov, B. Riecan, Intercriteria analysis of the intuitionistic fuzzy implication properties, Notes on Intuitionistic Fuzzy Sets, Vol. 21(5), 2015, 20–23.
5. P. Vassilev, L. Todorova, V. Andonov. An auxiliary technique for InterCriteria Analysis via a three dimensional index matrix. Notes on Intuitionistic Fuzzy Sets, 21(2), 2015, 71–76.
6. V. Traneva, V. Internal operations over 3-dimensional extended index matrices, Proceedings of the Jangjeon Mathematical Society, Vol. 18(4), 2015, 547–569.
7. D. Mavrov. Software for InterCriteria Analysis: Implementation of the main algorithm, Notes on Intuitionistic Fuzzy Sets, Vol. 21(2), 2015, 77–86.
8. D. Mavrov, I. Radeva, K. Atanassov, L. Doukovska, I. Kalaykov, InterCriteria Software Design: Graphic Interpretation within the Intuitionistic Fuzzy Triangle, Proceedings of the Fifth International Symposium on Business Modeling and Software Design, 2015, 279–283.
9. D. Stratiev, I. K Shishkova, A. Nedelchev, K. E. Kirilov, E. Nikolaychuk, A. S. Ivanov, I. Sharafutdinov, A. Veli, M. Mitkova, T. Tsaneva, N. Petkova, R. Sharpe, D. Yordanov, Z. Belchev, S. Nenov, N. Rudnev, V. Atanassova, E. Sotirova, S. Sotirov, K. Atanassov, Investigation of relationships between petroleum properties and their impact on crude oil compatibility, Energy & Fuels, American Chemical Society, 2015.
10. D. Stratiev, S. Sotirov, I. Shishkova, A. Nedelchev, I. Sharafutdinov, A. Vely, M. Mitkova, D. Yordanov, E. Sotirova, V. Atanassova, K. Atanassov, D. D. Stratiev, N. Rudnev, S. Ribagin Investigation of relationships between bulk properties and fraction properties of crude oils by application of the intercriteria analysis, Petroleum Science and Technology, Vol. 34, Issue 13, 2016, 1113–1120.
11. K. Atanassov. Intuitionistic Fuzzy Sets: Theory and Applications. Springer Physica-Verlag, Heidelberg, 1999.
12. S. Sotirov, V. Atanassova, E. Sotirova, V. Bureva, D. Mavrov, Application of the Intuitionistic Fuzzy InterCriteria Analysis Method to a Neural Network Preprocessing Procedure, 9th Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT), 30.06-03.07.2015, Gijon, Spain, 2015, 1559–1564.
13. T. Ilkova, M. Petrov, Application of InterCriteria Analysis to the Mesta River Pollution Modelling, Notes on Intuitionistic Fuzzy Sets, Vol. 21(2), 2015, 118–125.

14. T. Ilkova, O. Roeva, P. Vassilev, M. Petrov, InterCriteria Analysis in Structural and Parameter Identification of L-lysine Production Model, Issues in Intuitionistic Fuzzy Sets and Generalized Nets, Vol. 12, 2015, 39–52.
15. M. Angelova, O. Roeva, T. Pencheva, InterCriteria Analysis of Crossover and Mutation Rates Relations in Simple Genetic Algorithm, Annals of Computer Science and Information Systems, Vol. 5, 2015, 419–424.
16. O. Roeva, O., P. Vassilev, InterCriteria Analysis of Generation Gap Influence on Genetic Algorithms Performance, In: Novel Developments in Uncertainty Representation and Processing, Part V, (K. T. Atanassov, O. Castillo, J. Kacprzyk, M. Krawczak, P. Melin, S. Sotirov, E. Sotirova, E. Szmidt, G. De Tré, S. Zadrożny, Eds.), Vol. 401, Advances in Intelligent Systems and Computing, 2016, 301–313.
17. O. Roeva, S. Fidanova, M. Paprzycki, InterCriteria Analysis of ACO and GA Hybrid Algorithms, Studies in Computational Intelligence, Springer, Vol. 610, 2016, 107–126.
18. S. Fidanova, O. Roeva, M. Paprzycki, InterCriteria Analysis of Ant Colony Optimization Application to GPS Surveying Problems, Issues in Intuitionistic Fuzzy Sets and Generalized Nets, Vol. 12, 2015, 20–38.
19. T. Pencheva, M. Angelova, P. Vassilev, O. Roeva, InterCriteria Analysis Approach to Parameter Identification of a Fermentation Process Model, In: Novel Developments in Uncertainty Representation and Processing, Part V, (K. T. Atanassov, O. Castillo, J. Kacprzyk, M. Krawczak, P. Melin, S. Sotirov, E. Sotirova, E. Szmidt, G. De Tré, S. Zadrożny, Eds.), Vol. 401, Advances in Intelligent Systems and Computing, 2016, 385–397.
20. K. Atanassov. Intuitionistic fuzzy sets, VII ITKR's Session, Sofia, June 1983 (in Bulgarian).
21. K. Atanassov. Intuitionistic fuzzy sets. Fuzzy Sets and Systems. Vol. 20 (1), 1986, 87–96.
22. K. Atanassov. On Intuitionistic Fuzzy Sets Theory. Springer, Berlin, 2012.
23. L. A. Zadeh. Fuzzy Sets. Information and Control Vol. 8, 1965, 333–353.
24. K. Atanassov. Generalized Nets. World Scientific, Singapore, 1991.
25. K. Atanassov. Index Matrices: Towards an Augmented Matrix Calculus. Springer, Cham, 2014.
26. V. Atanassova, I. Vardeva. Sum- and average-based approach to criteria shortlisting in the InterCriteria Analysis. Int. J. Notes on Intuitionistic Fuzzy Sets, Volume 20(4), 2014, 41–46.
27. V. Atanassova, L. Doukovska, D. Karastoyanov, F. Capkovic. InterCriteria Decision Making Approach to EU Member States Competitiveness Analysis: Trend Analysis. P. Angelov et al. (eds.), Intelligent Systems'2014, Advances in Intelligent Systems and Computing 322, 107–115.
28. World Economic Forum. The Global Competitiveness Reports, 2008–2017. http://www.weforum.org/issues/global-competitiveness.
29. V. Atanassova, D. Mavrov, L. Doukovska, K. Atanassov. Discussion on the threshold values in the InterCriteria Decision Making approach. International Journal Notes on Intuitionistic Fuzzy Sets, Volume 20(2), 2014, 94–99.
30. V. Atanassova, L. Doukovska, K. Atanassov, D. Mavrov – InterCriteria Decision Making Approach to EU Member States Competitiveness Analysis, Proc. of the International Symposium on Business Modeling and Software Design – BMSD'14, 24–26 June 2014, Lux-embourg, Grand Duchy of Luxembourg, 2014, 289–294.
31. V. Atanassova, L. Doukovska, D. Mavrov, K. Atanassov. InterCriteria Decision Making Approach to EU Member States Competitiveness Analysis: Temporal and Threshold Analysis. P. Angelov et al. (eds.), Intelligent Systems'2014, Advances in Intelligent Systems and Computing 322, 95–106.
32. V. Atanassova. Interpretation in the Intuitionistic Fuzzy Triangle of the Results, Obtained by the InterCriteria Analysis, 16th World Congress of the International Fuzzy Systems Association (IFSA), 9th Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT), 30. 06-03. 07. 2015, Gijon, Spain, 2015, 1369–1374.
33. K. Atanassov. Geometrical Interpretation of the Elements of the Intuitionistic Fuzzy Objects, Mathematical Foundations of Artificial Intelligence Seminar, Sofia, 1989, Preprint IM-MFAIS-1-89. Reprinted: Int. J. Bioautomation, 2015, 19(4), Suppl. 2, S156–S171.

34. V. Atanassova, I. Vardeva, E. Sotirova, L. Doukovska. Traversing and Ranking of Elements of an Intuitionistic Fuzzy Set in the Intuitionistic Fuzzy Interpretation Triangle. In: Atanassov K. et al. (eds) Novel Developments in Uncertainty Representation and Processing. Advances in Intelligent Systems and Computing, Vol. 401. Springer, Cham, 2016.
35. K. Atanassov. Modal and topological operators, defined over intuitionistic fuzzy sets. In: Youth Scientific Contributions, Academic Publ. House, Sofia, 1, 1985, 18–21.
36. V. Atanassova, L. Doukovska, A. Michalikova, I. Radeva. Intercriteria analysis: From pairs to triples. Notes on Intuitionistic Fuzzy Sets, Vol. 22(5), 2016, 17–24.
37. L. Doukovska, V. Atanassova, G. Shahpazov, F. Capkovic, InterCriteria Analysis Applied to EU Micro, Small, Medium and Large Enterprises. In Proceedings of the Fifth International Symposium on Business Modeling and Software Design, Milan, Italy, 2015, 284–291.